BRIEF COMMUNICATIONS

DIFFUSION FLOW TO A MOVING DROPLET

V. V. Dil'man and B. B. Brandt

Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 5, pp. 662-665, 1967

UDC 532.72:532.526

The diffusion flow to a moving particle at small Reynolds numbers (Re ≪ 1) is described by a criterial equation of the form

$$\mathrm{Nu} \sim \mathrm{Pe}^m, \tag{1}$$

where m = 1/2 for a dissolving bubble and m = 1/3 for a dissolving solid particle [1].

In extraction the ratio of the dynamic viscosity of the liquid inside the droplet μ_1 to the viscosity of the external liquid μ_2 varies within the limits $0 < \mu = \mu_1/\mu_2 < \infty$. We will show that in expression (1) for the diffusion flow to a moving droplet the value of m also depends on μ. At small μ the exponent of the Pe number tends to 1/2, while at large μ it tends to 1/3.*

The stationary distribution of concentration C in a plane diffusion layer is described by the equation

$$\frac{dC}{dt} = v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}, \tag{2}$$

where t is the diffusion time and y the distance along the normal to the surface of the body (at the surface y = 0). The coordinate x is reckoned from the surface of the body in the direction of motion of the liquid. As Prandtl [3] has shown, the surface of a body may be considered plane if the thickness of the boundary layer is small as compared with the radius of curvature of the surface. The solution of Eq. (2) must satisfy the conditions

$$\begin{array}{l|l} x = 0 \\ (t = 0) & y > 0, \quad C = C_0, \\ L > x > 0 & y = 0, \quad C = 0, \\ (\tau > t > 0) & y \to \infty, \quad \dfrac{\partial C}{\partial y} \to 0. \end{array} \tag{3}$$

We introduce the following scales: time τ, distance along the normal to the surface λ, and concentration C_0, and reduce Eq. (2) and conditions (3) to the dimensionless form

$$\frac{dn}{d\xi} = \frac{D\tau}{\lambda^2} \frac{\partial^2 n}{\partial \zeta^2}; \tag{4}$$

$$\begin{array}{l} \xi = 0, \quad \zeta > 0, \quad n = 1; \\ 1 > \xi > 0 \left| \begin{array}{l} \zeta = 0, \quad n = 0 \\ \zeta \to \infty, \quad \dfrac{\partial n}{\partial \zeta} \to 0. \end{array} \right. \end{array} \tag{5}$$

Here, $n = C/C_0$, $\zeta = y/\lambda$, and $\xi = t/\tau$ are the dimensionless concentration, coordinate, and time.

Integration of Eq. (4) with conditions (5) gives

$$-\left(\frac{\partial n}{\partial \zeta}\right)_{\zeta=0} = \frac{\lambda^2}{D\tau} \int_0^\infty \frac{dn}{d\xi} d\zeta = \frac{\lambda^2}{D\tau} \Phi(\xi).$$

For the mean value of $\left(-\frac{\partial n}{\partial \zeta}\right)_{\zeta=0}$ with respect to ξ we have

$$-\left(\frac{\overline{\partial n}}{\partial \zeta}\right)_{\zeta=0} = \alpha \frac{\lambda^2}{D\tau}, \tag{6}$$

where α is some constant coefficient. We define the scale as the mean thickness of the diffusion layer for all t:

$$\delta = C_0 D/\bar{j},$$

where $\bar{j} = -D \frac{C_0}{\delta} \left(\frac{\overline{\partial n}}{\partial \zeta}\right)_{\zeta=0}$ is the mean value of the diffusion flux. Then $\left(\frac{\overline{\partial n}}{\partial \zeta}\right)_{\zeta=0} = -1$, and expression (6) takes the form

$$\delta = \beta \delta_E,$$

where $\delta_E = (2D\tau)^{1/2}$ is the mean displacement of the particles in time τ determined from the Einstein diffusion law; $\beta = 1/(2\alpha)^{1/2}$ is a proportionality factor. Since the dimensionless variables n, ξ, and ζ vary in the range from 0 to unity, an estimate of the proportionality factor gives

$$\beta = \frac{1}{\sqrt{2\int_0^1 \left(\int_0^\infty \frac{dn}{d\xi} d\zeta\right) d\xi}} \sim 1.$$

Thus, the problem of convective diffusion with a variable diffusion layer thickness $\delta(x)$ is reduced to the equivalent problem with a constant thickness δ. In this case the estimate of the characteristic time τ acquires special significance.

If the streamlines are parallel to the surface of the body and the fluid velocity U is not a function of the coordinates, the particles of diffusing substance carried by the flow into the layer δ are uniformly deposited over the entire surface of the body and on average the particle diffusion time

$$\tau = \frac{1}{L} \int_0^L t\,dx = \frac{1}{L} \int_0^L \frac{x}{U} dx.$$

Let the velocity U be an arbitrary function of the distance to the surface. As a result of the random motion a particle of diffusing substance will be at different distances from the surface at different moments of time and will be displaced by the flow along the body at different velocities. However, since the time during which the particle remains in a layer of thickness Δy does not depend on the value of the y coordinate, the average rate of displacement of the particle along the surface is equal to the average flow velocity with respect to y. In this case the particles of diffusing substance reach the surface on average after a time

$$\tau = \frac{1}{L} \int_0^L \frac{x}{\frac{1}{\delta}\int_0^\delta U(y)\,dy} dx.$$

If the velocity is a function of the coordinates x and y, the time in which particles of diffusing substance reach the surface of the body from the layer δ is determined only by the value of the tangential velocity component and can be estimated as

$$\tau = \frac{1}{L} \int_0^L \int_0^x \frac{dz}{\frac{1}{\delta}\int_0^\delta v_x(z, y)\,dy} dx.$$

*Similar arguments were put forward in [2].

The flow over a sphere of radius a is affected by the entire front part of the sphere. Therefore in calculating the time taken by the fluid to move along an arbitrary streamline in the layer δ it is necessary to integrate within the limits θ, $\pi-\theta$ (θ is the angle between the radius vector and the polar axis, which coincides with the direction of motion of the unperturbed flow; the coordinate origin is located at the center of the sphere).

The diffusion time averaged over the entire surface of the sphere

$$\tau = \frac{2}{4\pi a^2}\int\limits_0^{\frac{\pi}{2}} 2\pi a \sin\theta \int\limits_\theta^{\pi-\theta} \frac{d(a\psi)}{\frac{1}{\delta}\int\limits_0^\delta v_\theta(y,\psi)\,dy}\, d(a\theta). \quad (8)$$

We will estimate the diffusion flow to a spherical droplet moving at $Re \ll 1$. In accordance with the Hadamard-Rybchinskii solution, the tangential velocity

$$v_\theta = u\varphi\sin\theta, \quad (9)$$

where u is the velocity of the droplet;

$$\varphi = \frac{1}{2(1+\mu)} + \frac{1+3\mu}{2(1+\mu)}\,\frac{y}{a}.$$

From Eqs. (7)-(9) we obtain an equation for estimating the thickness of the diffusion layer:

$$\delta^2 = 4\beta^2\frac{Da}{u}\,\frac{1+\mu}{1+(1+3\mu)\frac{\delta}{2a}}\int\limits_0^{\frac{\pi}{2}}\sin\theta\int\limits_\theta^{\pi-\theta}\frac{d\psi}{\sin\psi}\,d\theta. \quad (10)$$

Integrating and keeping in mind that $Nu = a/\delta$, $Pe = ua/D$, we reduce Eq. (10) to the form

$$Pe = (16\ln 2)\,\beta^2(1+\mu)\frac{Nu^3}{1+3\mu+2Nu}. \quad (11)$$

For a bubble ($\mu \ll 1$, $Pe \gg 1$) expression (11) takes the form

$$Nu = \frac{0.42}{\beta}\,Pe^{\frac{1}{2}}. \quad (12)$$

If $\mu \to \infty$ (solid particle) and $Pe \gg 1$, Eq. (11) becomes

$$Nu = \frac{0.65}{\beta^{2/3}}\,Pe^{\frac{1}{3}}. \quad (13)$$

When the viscosities are comparable ($\mu \approx 1$), we have

$$Nu = \frac{0.42}{\beta}\,Pe^{\frac{1}{2}}(1+\mu)^{\frac{1}{2}}. \quad (14)$$

The numerical coefficients in (12)-(14) coincide with the exact values [1] if β is set equal to 0.91 in (12) and (14) and 1.02 in (13). This confirms the validity of the above estimate of β and indicates that this coefficient depends only very slightly on viscosity.

It may be assumed that the function $Nu(\mu)$, determined from Eq. (11), does not have singularities in the interval of variation of viscosity $0 \le \mu \le \infty$. Therefore the quantity β in Eq. (11) may be taken equal to its mean value of 0.97. When the above value of the coefficient β is employed, the diffusion flux is calculated correct to approximately ±6%.

Since we have assumed the stationarity of the concentration field, the method proposed is applicable for times t much greater than the time τ during which as a result of diffusion to the surface of the body the particles are displaced through a distance equal to the thickness of the diffusion layer δ, i.e.,

$$t \gg \tau = \frac{\delta^2}{2\beta^2 D}.$$

In the stationary regime the diffusion time is equal to the convection time. The latter is equal in order of magnitude to a/u, which makes it possible to put the stationarity condition in the form $t \gg a/u$, a form convenient for practical calculations.

NOTATION

a is the radius of droplet (sphere); C is the concentration, C_0 is the same remote from the surface; D is the diffusion coefficient; j is the diffusion flux; $n = C/C_0$ is the dimensional concentration; t is time; U and u are the velocity of liquid and droplet, respectively; $v_\theta v_x$ is the tangential component of velocity; v_y is the normal component of velocity; x is the coordinate along surface of body; y is the coordinate along normal to surface; $Nu = a/\delta$ is the Nusselt number; $Pe = ua/D$ is the Peclet number; Re is the Reynolds number; $\beta = 1/(2\alpha)^{1/2}$ is a coefficient; δ is the thickness of diffusion layer; $\zeta = y/\lambda$ is a dimensionless coordinate; ψ, θ are polar angles; λ is the scale in the y direction; μ_1, μ_2 is the dynamic viscosity of liquid inside and outside droplet, $\mu = \mu_1/\mu_2$; $\xi = t/\tau$ is dimensionless time; τ is the time scale.

REFERENCES

1. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, 1959.
2. O. E. Potter, Chem. Eng. Sci., 6, 170, 1957.
3. L. Prandtl, Hydroaerodynamics [Russian translation], IL, 1949.

6 September 1966

Institute of the Nitrogen Industry and Products of Organic Synthesis, Moscow

CALCULATION OF THE TEMPERATURE FIELD IN THE WALL OF A SEGMENTED-ELECTRODE MHD CHANNEL ON AN EHDA-9/60 INTEGRATOR

O. Ya. Kazakevich

Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 5, pp. 666-668, 1967

UDC 536.212:533.9.07

It is now becoming clear that the use of ceramic materials in the channels of MHD generators operating on combustion products at 2500-3000° K must be extremely limited. The experimentally observed damage and the interaction of the plasma with the duct walls exclude the possibility of using uncooled systems over extended periods, but it is to be expected that water-cooled metal walls will be suffi-